ANALYTICITY AND hp DISCONTINUOUS GALERKIN APPROXIMATION OF NONLINEAR SCHRODINGER EIGENPROBLEMS

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Résumé

We study a class of nonlinear eigenvalue problems of Schrödinger type, where the potential is singular on a set of points. Such problems are widely present in physics and chemistry, and their analysis is of both theoretical and practical interest. In particular, we study the regularity of the eigenfunctions of the operators considered, and we propose and analyze the approximation of the solution via an isotropically refined hp discontinuous Galerkin (dG) method.

We show that, for weighted analytic potentials and for up-to-quartic nonlinearities, the eigen-functions belong to analytic-type non homogeneous weighted Sobolev spaces. We also prove quasi optimal a priori estimates on the error of the dG finite element method; when using an isotropically refined hp space the numerical solution is shown to converge with exponential rate towards the exact eigenfunction. In addition, we investigate the role of pointwise convergence in the doubling of the convergence rate for the eigenvalues with respect to the convergence rate of eigenfunctions. We conclude with a series of numerical tests to validate the theoretical results.

Mots-Clés: Analyse numérique, méthode des éléments finis, adaptativité, problème aux valeurs propres non linéraire, problèmes de type Schrödinger

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